Statistical Methods in Theory of Technical Systems

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Abstract

Today's society is served by modern technology, accompanied by effects of insecurity and uncertainty, which are important and inevitable. Engineering analysis must include methods of assessing these impacts on the design, implementation and operation of technical systems. Mathematical statistics and probability theory provide the mathematical basis for modeling unreliability and analysis as well as their impact on already built technical systems. The aim of the paper is to introduce quantitative methods that enable systematic development of planned and constructed criteria, problem modeling and evaluation of economically technical optimum.

Keywords: mathematical statistics, quantitative reliability calculations, computer simulations

1. Introduction

Every technical system is designed, built and implemented to perform the specific function, determined by the demands of the system users. Planning and building of the technical systems is followed by the influence of the uncertainty and unreliability, which is the reason why the engineering analysis must include estimation methods of these influences on construction, build and performance of technical systems [1], [2].

System errors can occur because of the physical or chemical reasons, or errors caused by the human factor. Engineers are trying to show the operational role of the systems and increase their lifetime, eliminating or minimizing the security risks. Availability of the measurements includes also important system characteristics [3], [4].

Quantitative calculation of reliability, availability and security is a newer scientific discipline [5], [6], [7]. The aim of this paper is to present quantitative methods that include mathematical modeling [8] and computer simulations in engineering practice [9], [10]. Mathematical statistics and probability theory give the mathematical foundations to model uncertainty and analysis, and it's influence on already built technical systems [11], [12]. [13].

They enable systematic development of planned and constructed criteria, problem modeling and estimation of economically technical optimal solutions [14].

2. Reliability theory methods

The need of analyzing reliability, availability and security of technical systems is increasing over the last years. It demands precise definition of the terms and the development of the reliability theory mathematical tools.

2.1. Statistical methods

Mathematical statistics answers the question whether the model, proposed by the empiricists, describes the appropriate random trials, while the probability theory calculates probabilities of the different random events (such as failures, accidents.) according to the given model [15].

In operation of some technical systems, failures occur successively and completely randomly, which represents a random process. The measure of reliability of the component or system is the frequency of the failure occurrences in time. Relative frequency of a random event (such as failure of component) depends on the number of trials in a row, so we do not know which one of these frequencies is actually the probability. But, experience shows that relative frequency along with appropriate conditions can serve as an estimator for the exact probability [16].

Mathematical statistics gives us the exact methods how to determine the frequencies of the occurrence of the events that can be described by statistical distributions. From these frequencies the probability of the observed events is calculated. To answer the question how to determine the probability of some random event, we use combinatorics and probability theory [17].

2.2. Computer simulations

When the distribution of failures is determined, the probability of occurrence of the failures of the components (system, for example, due to wearout) is mathematically calculated. Computer simulations of the distribution functions [9] make a very important part of the research, because they give an insight on the behaviour of the problem (system), especially if the experiment is expensive [9]. Figures 1 and 2 shows PDF of

Normal and Exponential distributions plotted using computer package Matlab (Matlab: → Toolboxes→Statistics→ disttool).

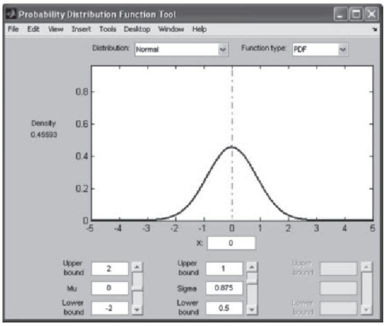

Figure 1: Normal distribution[9].

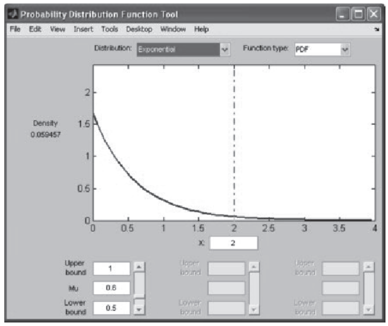

Figure 2: Exponential distribution[9].

2.3. Quantitative methods

Quantitative methods are based on numerical estimation of the probability of the failure occurrence, where the parameters are quantified by statistical methods and databases. [18], The methods enable determining of the reference numerical data, i.e. known reliability measures of each system component (such as mean time to failure,MTTF; failure rate , λ (t)), in order to evaluate reliability of the system. If the frequency of the failures of the working component is constant, it is suitable to use a homogeneous Markov chain for the mathematical baseline to model the reliability of such system. [2], [19]. [20].

The reliability and availability analysis of technical systems using Markov model is one of the reliability theory quantitative methods [21], [22], As an example was chosen ideal non-repair two components standby system [1]. A backup system can be compared to a parallel system, where we assume the components represent for ex. two transformers. In a parallel system both transformers will work from start until the failure occurrence. While in standby system, a commutator will incorporate only one transformer and the second will stand as a backup. The second will be introduced at the moment of failure and the operation will continue with no interruption [2], [5]. An ideal non-repair two-component standby system is taken as an example.

2.3.1. Example: Determination the reliability of ideal non-repair two components standby system by Markov model

Let us look at the ideal non-repair two components standby system and let's suppose that the both components, when they are in operation, have constant failure rate, say λ.

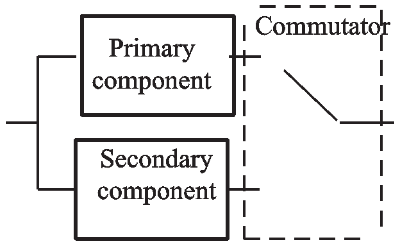

Figure 3. Diagram of ideal non-repair two components standby system [1].

System state is determined with the states of its components. So, the states of the system show whether the entire system is in operational or in failure state. The crossings occur when the component or(and) the system changes state. Since the system consists of the components, the crossings occur due to the change of the state of the component. The change of the state of the component can be caused by failure or the repair of the component.

In order to apply the Markov process, it is necessary to define the characteristic states of the system, which are also the states of the Markov process.

For the given system, the states are:

State 1: the primary component is operational, and the secondary component is standby

State 2: the primary component has failure and the secondary is operational

State 3: both components have failure

In states specified like that, q_{12} is failure rate of the primary component q_{23} is failure rate of the secondary component. The observed system has the following transition matrix Q:

$$Q=\begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix}=\begin{bmatrix} -\lambda & \lambda & 0 \\ 0 & -\lambda & \lambda \\ 0 & 0 & 0 \end{bmatrix} \tag{1}$$

The associated Kolmogorov system of differential equations is

$$\left(p'_1(t), p'_2(t), p'_3(t)\right)=\left(p_1(t), p_2(t), p_3(t)\right)\cdot\begin{bmatrix} -\lambda & \lambda & 0 \\ 0 & -\lambda & \lambda \\ 0 & 0 & 0 \end{bmatrix} \tag{2}$$

Reliability of the system $R_r(t)$ is determinanted by the sum of the probabilities of the state 1 and the state 2:

$$R_r(t)=p_1(t)+p_2(t) \tag{3}$$

Since the observed system is non-repair, state 3 is absorbing. That is why it is enough to use the reduced transition matrix Q^* to determine the probabilities of the states $p_1(t)$ and $p_2(t)$:

$$Q^* = \begin{bmatrix} -\lambda & \lambda \\ 0 & -\lambda \end{bmatrix} \tag{4}$$

Then the probabilities of the states at the moment t, $p_1(t)$ and $p_2(t)$, are the solutions of the system of differential equations given in the form [6]:

$$\left(p'_1(t), p'_2(t)\right) = \left(p_1(t), p_2(t)\right) \cdot \begin{bmatrix} -\lambda & \lambda \\ 0 & -\lambda \end{bmatrix} \tag{5}$$

that is

$$\begin{aligned} & p'_1(t) + \lambda p_1(t) = 0 \\ & p'_2(t) - \lambda p_1(t) + \lambda p_2(t) = 0 \end{aligned} \tag{6}$$

If we use the initial conditions $p_1(0)= 1$ and $p_2(0)= 0$ the Laplace transforms [10] may be used to solve equations (7):

$$\begin{aligned} & (s+\lambda) P_1(s) = 1 \\ & -\lambda P_1(s) + (s+\lambda) P_2(s) = 0 \end{aligned} \tag{7}$$

We have:

$$\begin{aligned} & P_1(s) = \frac{1}{s+\lambda} \\ & P_2(s) = \frac{\lambda}{(s+\lambda)^2} \end{aligned} \tag{8}$$

Now, applying the inverse Laplace transforms, we get:

$$\begin{aligned} & p_1(t) = e^{-\lambda \bullet t} \\ & p_2(t) = \lambda t e^{-\lambda t} \end{aligned} \tag{9}$$

Summing these two probabilities we get the expression that describes the reliability of the system:

$$MTTF_r = \int_0^\infty \left(e^{-\lambda t} + \lambda t e^{-\lambda t}\right) dt = \frac{1}{\lambda} + \frac{1}{\lambda} = 2\frac{1}{\lambda} = 2MTTF \tag{10}$$

The mean time to failure, is given by the following expression

$$R_r(t) = (1+\lambda t) \cdot e^{-\lambda t} \tag{11}$$

where MTTF_c is the mean time to failure of each of the components in operation mode.

3. Results

A standby system is chosen as an example of systems with interdependent system components. From the expression (11) we can conclude that the mean time to failure of an ideal non repair two component system with a reserve is equal to the sum of the mean time to failure of its components, when they are in operation. As given in the example, the stand by system can be compared to the parallel system and reliabilities can be calculated in both systems. In standby system there is always conditional probability because the second component does not start to operate if the first one doesn't stop. Therefore, reliabilities of those systems will not match.

4. Discussion

In the analysis of the reliability of technical systems, quantitative methods enable modeling of the compound relations in a system, as well as modeling of the system reliability dynamics. If the failure rate of each of the components, when in operation, is constant, it is suitable to use a homogeneous Markov process for modeling reliability of such a system. The components of such system are interdependent, since failure rate of the components in backup rapidly changes from zero to a positive value in the moment of its automatic operationality due to failure of the earlier operational component. The application of quantitative methods is visible in the project Quantitative methods in function of the optimal management of the maritime system carried out by the Ministry of Science, Education and Sports. [23].

5. Conclusion

Quantitative estimation of the reliability of technical systems makes the development of the mathematical models which describe connections between the system components and effect on reliability, i.e. the probability of successful system operation, possible.

The use of techniques of modeling the reliability, such as Markov analysis, should point out the problems in the process of collecting data on components failures, i.e. the system failures, and propose adequate measures for upgrading its reliability.

The applications in maritime technology can be made in estimating reliability of the ship's diesel engines, the control and organization of maritime transport and the risk assessment.

Commercially available reliability databases are published in OREDA, Reliability *Data Handbook*. Further research should point on all the problems and defects in data collecting process of the components and system failure on a ship and propose the measures to increase its reliability.

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